

LABORATOIRE DE MATHÉMATIQUES  
UNIVERSITÉ PARIS-SACLAY, UMR 8628

## Partial Differential Equations in Biology

*le lundi 18 juillet 2022 au Bâtiment 307, Amphi Yoccoz*  
*Journée organisée par*  
*Danielle Hilhorst et Yong Jung Kim*

<https://zoom.us/j/96243145881?pwd=WHdkRXZZOGFRVmhvcElKakFOQTJlQT09>

**10h00 - 10h40, Yong Jung KIM (KAIST)**

Modeling bacterial traveling wave patterns

**10h45 - 11h25, Thomas GILETTI (Université de Lorraine)**

Propagation in a shifting environment

**11h45 - 12h25, Jaekyoung KIM (KAIST)**

Analysis of time series data with hidden components

**14h20 - 14h40, Kyunghan CHOI (KAIST)**

Turing pattern from chemotaxis cell aggregation model with signal-dependent motility

**14h45 - 15h05 Théo BELIN (CentraleSupélec)**

An asymptotic strict comparison principle for the one and two-phase Stefan problem

**15h10 - 15h50 Min-Gi LEE (Kyungpook National University)**

$L^2$  stability of strong solution of Euler-Poisson system in 1 space dimension

**16h20 - 17h00 Grégory FAYE (Université Paul Sabatier)**

The logarithmic Bramson correction for Fisher-KPP equations on the lattice  $Z$

## *Titles and abstracts*

### **Théo BELIN**

#### *An asymptotic strict comparison principle for the one and two-phase Stefan problem*

Abstract: Free boundary problems, such as the Stefan problem, constitute a class of non-linear problems modelling the interaction between two heterogeneous phases. Typically the Stefan problem describes the melting and freezing of water without convection. Free boundary problems are known to generate challenging geometrical evolutions : topological changes, formation of singularities, fattening ... A way to tackle these problems is via viscosity solutions. First developed by Oskar Perron in 1923, they became very popular in the 1990 for solving non-linear PDEs and geometric problems. These include successful treatment of the mean curvature flow, the Stefan problem, and more generally Hamilton-Jacobi equations. The viscosity theory relies on so-called comparison principles, which aim to provide a comparison of sub and supersolutions on the whole domain, only knowing comparison on the boundary. In my talk, I will present a strict comparison principle for the Stefan problem which refines an already existing result by Kim and Pozar. I show that the distance between the free boundaries of two solutions, starting at well prepared and strictly separated data, must be bounded from below.

### **Kyunghan CHOI**

#### *Turing pattern from chemotaxis cell aggregation model with signal-dependent motility*

Abstract: In this talk, the pattern formation of a chemotaxis model with Fokker-Planck type diffusion with the motility dependent on signal is presented. We do the stability analysis of the model and present criteria for pattern formation. The critical cell density that determines the cell aggregation phenomenon is obtained. We observe through numerical simulations the three Turing patterns from the model together with the classical chemotactic cell aggregation pattern. The Turing patterns feature the phase transition layer that can be seen in the framework of Cahn-Hilliard(CH) equation. Using the Van der Waals-Cahn-Hilliard theory, we investigate the steady-state problem of the aggregate model to find out why the phase-transition phenomenon happens. Furthermore, we verify that the motility that gives the unusual pattern is natural from a modeling point of view by limiting two PDE systems.

### **Grégory FAYE**

#### *The logarithmic Bramson correction for Fisher-KPP equations on the lattice $Z$*

Abstract: In this talk, we will report on some recent work regarding the logarithmic Bramson correction for Fisher-KPP equations on the lattice  $Z$  where we prove that the level sets of solutions with step-like initial conditions are located at position  $c_*t - \frac{3}{2\lambda_*} \ln t + \mathcal{O}(1)$  as  $t \rightarrow +\infty$  for some explicit positive constants  $c_*$  and  $\lambda_*$ . This extends a well-known result of Bramson in the continuous setting to the discrete case using only PDE arguments. This is joint work with C. Besse, J.-M. Roquejoffre and M. Zhang.

## Thomas GILETTI

### *Propagation in a shifting environment*

Abstract: In this talk we will investigate the large time behavior of solutions of reaction-diffusion equations which include a shifting heterogeneity, i.e. depending on a moving variable  $x - ct$  with  $c \geq 0$ . Such problems arise in the modeling of population dynamics ongoing a change in environmental conditions (climate change, invasive predator species...). We will tackle two situations depending on whether the reaction or the diffusion is heterogeneous. The results that I will present have been obtained in collaborations with Juliette Bouhours, Gregory Faye and Matt Holzer.

## Jaekyoung KIM

### *Analysis of time series data with hidden components*

Abstract: Despite dramatic advances in experimental techniques, many facets of intracellular dynamics remain hidden, or can be measured only indirectly. In this talk, I will describe three strategies to analyze timeseries data from biological systems with hidden parts: replacement of hidden components with either time delay, quasi-steady-state or random regulatory process. Then, I will illustrate how the simplification with the time delay can be used to understand the processes of protein synthesis, which involves multiple steps such as transcription, translation, folding and maturation, but typically whose intermediates proteins cannot be measured. Furthermore, I will illustrate how the simplification with the quasi-steady-state can be used to develop an accurate method to estimate drug clearance, which occurs in multiple steps of metabolism, which greatly improved the canonical approach used in more than 65,000 published papers for last 30 years. Finally, I will describe a systematic modeling selection approach to identify hidden regulatory biochemical connections leading to the observed timeseries data. Then, I will illustrate how we applied the approach to find the connection between the circadian clock and cell cycle checkpoints.

## Yong Jung KIM

### *Modeling bacterial traveling wave patterns*

Abstract: We develop a chemotaxis model with population growth in the paper. The developed mathematical models are based on the conversion dynamics between active and inactive cells with different dispersal rates. The process consists of three steps and the performance of each step is complemented by comparing numerical simulations and experimental data.

## Min-Gi LEE

### *$L^2$ stability of strong solution of Euler-Poisson system in 1 space dimension*

Abstract: We consider the Euler-Poisson system, where the force consists of the pressure gradient and the term depending non locally on the density. This system can be treated as a system of conservation laws that admits an additional conservation law of hamiltonian energy. As the system is formally a hamiltonian flow, the classical technique of relative entropy can be applied to have the  $L^2$  stability of strong solution. This has been done by Giesselmann, Lattanzio, and Tzavaras. This stability result typically is derived from a Gronwall type inequality, more specifically,  $L^2$  distance square at a finite time  $T$  is bounded by that of initial time multiplied by a constant but this constant exponentially grows with time.

Since the system is about a compressible motion of a certain material system, one can formally consider a formulation in Lagrangian coordinate system. What is more relevant is a choice of state variable. In the description in Eulerian coordinate system, the density and the velocity (or the momentum) are taken as the state variables. We instead take the position map and the momentum as the state variables. We think it more natural in a sense that in a classical particle mechanics, Newton's second law is formulated as an hamiltonian system with the position and momentum as a conjugate pair of state variables. We show that this choice in Lagrangian coordinate system results in non-increasing  $L^2$  distance between two states in one space dimension, which is much stronger estimate than that derived from Gronwall type estimate. We also present the difficulty in extending the result in multi dimensions. This is joint work with Kiwoong Kwon and Jan Giesselmann.