A ReaDiNet workshop on reaction-diffusion systems and population dynamics

June 10th to 14th - VVF Montpeyroux Parent

Time	Monday	Tuesday	Wednesday	Thursday
9:00 - 9:50	Yong-Jung KIM	Chueh-Hsin CHANG	Baptiste MAUCOURT	Samuel TRETON
9:50 - 10:20	*Coffee break*	*Coffee break*	*Coffee break*	*Coffee break*
10:20 - 11:10	Léo GIRARDIN	Amic FROUVELLE	Nicolae CINDEA	Cécile TAING
11:10 - 12:00	Quentin GRIETTE	Antoine DIEZ	Min-Gi LEE	Ho-Youn KIM
12:00 - 14:00	*Lunch*	*Lunch*	*Lunch*	*Lunch*
14:00 - 14:50	Matthieu ALFARO	Ken-Ichi NAKAMURA	*Free afternoon*	Arnaud DUCROT
14:50 - 15:40	Harunori MONOBE	Idriss MAZARI		Luca ALASIO
15:40 - 16:10	*Coffee break*	*Coffee break*		*Coffee break*
16:10 - 17:00	Elisa AFFILI	Pierre GABRIEL		Toshiyuki OGAWA
17:00 - 17:50	Junsik BAE	Sheila Rae PERMANES		

ABSTRACTS

Elisa AFFILI

A mathematical model for civil wars: a new Lotka-Volterra competitive system

Imagine two populations sharing the same environmental resources in a situation of open hostility. The interactions among these populations are governed not by random encounters but via the strategic decisions of one population that can attack the other according to different levels of aggressiveness. This leads to a non-variational model for the two populations at war, taking into account structural ecological parameters. We present the strategies that lead to the victory of the aggressive population, i.e., the choices of the aggressiveness parameter, in dependence of the structural constants of the system and possibly varying in time, which take to the extinction in finite time of the defensive population. The model that we present is flexible enough to include also technological competition models of aggressive companies releasing computer viruses to set a rival company out of the market. This is a joint work with S. Dipierro, L. Rossi and E. Valdinoci.

Luca ALASIO

Mathematical models for the visual cycle and retinal degenerations

The visual cycle is a fundamental bio-chemical process in the retina: it allows photoreceptors to convert light into electrical and chemical signals (phototransduction), before returning to the dark state. George Wald obtained the Nobel Prize in 1967 for his pioneering studies on this process and it has been an active field of research in Ophthalmology ever since. I will introduce the main aspets of the visual cycle and present a mathematical model for the concentrations of vitamin A isomers and related enzymes (involving coupled ODEs and PDEs). The goal is to give a quantitative description of the kinetics of the main photosensitive molecules after exposure to light. I will explain connections with the accumulation of toxic byproducts in the eye and with degenerative retinal diseases. In collaboration with M. Paques, Y. Borella, L. Almeida and B. Perthame, we are also studying the retinal pigment epithelium (RPE), which supports photoreceptors and participates in the visual cycle. We are investigating different mathematical models describing RPE in the presence of the age-related macular degeneration (AMD).

Matthieu ALFARO

A host-pathogen coevolution model

We propose a novel model describing the coevolution between hosts and pathogens, based on a non-local partial differential equation formalism for populations structured by phenotypic traits. Our objective with this model is to illustrate scenarios corresponding to the evolutionary concept of "Chase Red Queen scenario", characterized by perpetual evolutionary chases between hosts and pathogens. This is an ongoing work with F. Lavigne and L. Roques.

Junsik BAE

Nonexistence of multi-dimensional solitary waves in unmagnetized plasma

We study the nonexistence of multi-dimensional solitary waves for the Euler-Poisson system governing ion dynamics in plasma. It is well-known that the one-dimensional Euler-Poisson system has solitary waves travel faster than the ion-sound speed. In contrast, we show that the two-dimensional and three-dimensional models do not admit nontrivial irrotational spatially localized traveling waves for any traveling velocity and for general pressure laws. Our finding is strong evidence for the transverse stability of line solitary waves in the multi-dimensional Euler-Poisson system. We derive some Pohozaev type identities associated with the energy and density integrals. This approach is extended to prove the nonexistence of irrotational multi-dimensional solitary waves for the two-species Euler-Poisson system for ions and electrons. This is a joint work with Daisuke Kawagoe.

Chueh-Hsin CHANG

The bifurcation points of front-back-pulse solutions in the three-species Lotka-Volterra competition-diffusion system

In this talk we consider the existence of bifurcation points of the glued heteroclinic orbits in the Lotka-Volterra competition-diffusion system. The glued heteroclinic orbits represent the traveling wave solutions with profiles like front, back, and pulse for the three species, hence called front-back-pulse (FBP) solutions. We proved the existence of FBP solutions for parameters lying on a hypersurface near the bifurcation points. We give some examples of bifurcation points that can be expressed explicitly or numerically based on the previous work on two-species traveling waves. This is joint work with Chiun-Chuan Chen, Shin-Ichiro Ei, Toshiyuki Ogawa and Takashi Teramoto.

Nicolae CINDEA

Mathematical modelling and numerical simulation of chemical reactions during hemodialysis

The aim of this talk is to present a mathematical model and its numerical simulation for the evolution of the concentrations of several chemical species in a dialyzer during haemodialysis using a citrate dialysate. The model is composed of two parts. The first one describes the flows of blood and dialysate in a dialyzer fibre. It was obtained by asymptotic analysis and takes into account the anisotropy of the fibres forming a dialyzer. Newtonian and non-Newtonian blood rheologies were tested. The second part of the model predicts the evolution of the concentration of five chemical species present in these fluids. The fluid velocity field drives the convective part of a convection–reaction–diffusion system that models the chemical reactions. We performed several numerical experiments to calculate the free calcium concentration in the blood in a dialyzer using dialysates with or without citrate. The model was validated by a clinical study conducted on 22 patients. Finally, we discuss a parameter optimization strategy in order to better fit the experimental results and we give some perspective for an optimal control problem concerning the chemical composition of the dialysate fluid.

Antoine DIEZ

Understanding limb morphogenesis: from 3D organoids to mathematical models

Despite the experimental advances in controlling cell fate to obtain various cell types in culture, it is still challenging to produce tissues with a certain shape, especially those made of mesenchymal cells such as limbs. Limb morphogenesis is an emerging phenomenon which can be observed in 3D organoid cultures: from a spherical aggregate of cells, one or several limb buds spontaneously develop through various complex chemical and mechanical processes. Experimental imaging data give a good insight on the individual and collective behavior of the cells but the precise interaction mechanisms leading to the emergence of limb buds remain unclear. With a first principle approach, we propose a mathematical model based on a novel coupling hypothesis between the dynamics of morphogens and the mechanical forces acting between the individual cells. Numerical simulations in a simplified setting show the emergence of an arbitrary number of limb buds. The mathematical analysis leads to a Cahn-Hilliard model with extra terms which are believed to play a crucial role in the symmetry breaking. Understanding the sharp interface limit associated to this model is a challenging problem which is still under investigation. This is a joint work (in progress) with Rio Tsutsumi, Steffen Plunder and Hyunjoon Park.

Arnaud DUCROT

Spreading property for prey-predator systems on a lattice

We investigate the spreading behaviour for the solutions of a prey-predator system on a discrete lattice. The spatial motion of individuals from one site to another is modeled by a discrete convolution operator. Our analysis of the spreading speeds of invasion of the species is based on the careful and detailed study of the hair-trigger effect and spreading speed for a non-autonomous scalar Fisher-KPP equation on a lattice. Then, we are able to compare the solutions of the prey-predator system with those of a suitable scalar Fisher-KPP equation and derive the invasion speeds of the prey and of the predator.

Amic FROUVELLE

Hypocoercivity in a model of aligning self-propelled particles

This talk is about a joint work with Emeric Bouin. We are interested in the long-time behavior of the kinetic Vicsek equation with phase transition, in its spatially localized version. The method follows the now-classic approach of modifying Sobolev-type norms by adding cross-terms, linked to commutators between the different operators appearing in the kinetic equation. However, the fact that the velocity space is the sphere adds significant subtleties and requires to develop an adapted algebraic framework of operators. Taking advantage of this new framework, we manage to perform an approach à la Hérau to show the nonlinear stability.

Pierre GABRIEL

On the principal eigenvalue problem for positive semigroups

We revisit the Krein-Rutman theory for semigroups of positive operators, having in mind the application to linear PDEs which preserve positivity. We propose simple and applicable criteria for ensuring the existence, uniqueness, and (quantified) exponential stability of the principal eigentriplet made of the first eigenvalue and its associated primal and dual eigenvectors. The method can be applied to parabolic PDEs, kinetic transport equations, kinetic Fokker-Planck equations, growth-fragmentation equations, or purely integral equations. We will detail some of these examples. It is a joint work with Claudia Fonte Sanchez and Stéphane Mischler.

Leo GIRARDIN

Persistence and propagation of structured populations in space-time periodic media

This talk presents recent results on asymptotic persistence, extinction and spreading properties for structured population models resulting in non-cooperative Fisher-KPP systems with space-time periodic coefficients. Results are formulated in terms of a family of generalized principal eigenvalues associated with the linearized problem. When the maximal generalized principal eigenvalue is negative, all solutions to the Cauchy problem become locally uniformly positive in long-time, at least one space-time periodic uniformly positive entire solution exists, and solutions with compactly supported initial condition asymptotically spread in space at a speed given by a Freidlin-Gärtner-type formula. When another, possibly smaller, generalized principal eigenvalue is nonnegative, then on the contrary all solutions to the Cauchy problem vanish uniformly and the zero solution is the unique space-time periodic nonnegative entire solution. When the two generalized principal eigenvalues differ and zero is in between, the long-time behavior depends on the decay at infinity of the initial condition. The proofs rely upon double-sided controls by solutions of cooperative systems. The control from below is new for such systems and makes it possible to shorten the proofs and extend the generality of the system simultaneously.

Quentin GRIETTE

Speed-up of traveling waves by negative chemotaxis

We consider the traveling wave speed for Fisher-KPP (FKPP) fronts under the influence of chemotaxis and provide an almost complete picture of its asymptotic dependence on parameters representing the strength and length-scale of chemotaxis. Our study is based on the convergence to the porous medium FKPP traveling wave and a hyperbolic FKPP-Keller-Segel traveling wave in certain asymptotic regimes. In this way, it clarifies the relationship between three equations that have each garnered intense interest on their own. Our proofs involve a variety of techniques ranging from entropy methods and decay of oscillations estimates to a general description of the qualitative behavior to the hyperbolic FKPP-Keller-Segel equation. For this latter equation, we, as a part of our limiting arguments, establish an explicit lower bound on the minimal traveling wave speed and provide a new construction of traveling waves that extends the known existence range to all parameter values. This is a joint work with Chris Henderson and Olga Turanova.

Ho-Youn KIM

Uniqueness of weak solutions for the Maxwell-Stefan system

The Maxwell-Stefan system describes diffusion phenomena in multicomponent gaseous fluids. For the existence problem, the local-in-time strong solution and the global-in-time weak solution were studied, while for the uniqueness problem, only the uniqueness of the strong solution and the weak-strong uniqueness property were proven. In this work, we show that the weak solution to the Maxwell-Stefan system is unique, and the proof is based on the dissipation of the symmetrized relative entropy.

Yong-Jung KIM

Diffusion can provide spatial separation to epidemic models

We propose an SIS reaction-diffusion epidemic model and show the effect of migration strategies on disease transmission when the environment is spatially heterogeneous. We will see that infected and healthy people behave differently and that may segregate the two groups and reduce the basic reproduction number $\mathcal{R}_0$. This may give a new use of heterogeneous diffusion theory to epidemic models.

Min-Gi LEE

On fractioned traveling wave solutions of viscous coupled Burgers' equations

In this talk, we consider the system of viscous coupled Burgers' equations in one spatial dimension. It is well-known that in the scalar Burgers' equation, an admissible traveling shock wave is obtained as the limit of the traveling wave solution of the viscous equation, known as the viscous shock. The discontinuity of such an admissible shock, which is nonnegative, is always a downward jump discontinuity in the direction of travel. In coupled Burgers' equations, researchers have primarily examined special traveling wave solutions that are pairs of these typical downward-stepping profiles, traveling in the direction of jumping down. However, in the system case, a variety of traveling wave patterns emerge, which should not come as a surprise. Among these patterns, we characterize traveling waves where one species is monotonically increasing and the other species is monotonically decreasing. We refer to this as a fractioned traveling wave in this talk. Interestingly, every such fractioned traveling wave is obtained exclusively through a saddle-unstable connection and corresponds only to the 1-shock of the inviscid hyperbolic system.

Baptiste MAUCOURT

An optimization problem on the time integral of the solution of a semilinear parabolic-ordinary differential system modeling the sugar-beet agroecological system

In this presentation, we provide a biological context for the sugar-beet agroecological system and discuss its modeling. We explore aspects of an optimization problem arising from our model, which involves a semi-linear parabolic-ordinary differential system and a quantity called harvest. Using sub and supersolutions, we derive estimates for this quantity. We investigate the spatial homogenization of a parameter that optimizes the principal eigenvalue of the operator of the linearized system, but may not necessarily optimize the linearized harvest depending on the chosen initial conditions. We propose conjectures regarding the spreading speeds of solutions in the event of pest invasion in the field. Additionally, we examine an example of a linearized yield optimizer for a specific class of initial conditions: strictly decreasing rearranged Schwarz functions and constants. Finally, we present several numerical simulations to validate our theoretical findings.

Idriss MAZARI

Long-time behaviour for fishing systems: monotonicity and weak KAM theory

In this talk, I will present a recent preprint in collaboration with Z. Kobeissi and D. Ruiz-Balet. In it, we investigate a fishing problem where fishermen are competing so as to maximise their output. The model consists of a coupling between a Mean Field Game system and reaction-diffusion equations. We analyse the qualitative properties of this system, with a special emphasis on uniqueness and long-time behaviour of solutions. This latter part relies on the weak KAM framework put forth by Cardaliaguet in the context of Mean Field Games.

Harunori MONOBE

Singular limit of mathematical models related to controlling invasive alien species

We consider the dynamics of some Lotka-Volterra competition reaction-diffusion systems with the effect of controlling species. Our purpose is to investigate how the species behave depending on the effect of controlling species is large. In particular, we study the singular limit of the model and show that some free boundary problems appears.

Ken-Ichi NAKAMURA

The speed of bistable traveling fronts in the Lotka-Volterra competition-diffusion system

We consider front propagation in the classical 2-species Lotka-Volterra competition-diffusion system under strong competition conditions. The system has a unique traveling front solution (up to translation) connecting two stable states. The sign of the front speed gives us significant information about which species prevails over the other, and identifying the sign is still a challenging problem. In this talk, we give some new results on the sign of the speed of bistable traveling fronts based on comparison arguments. The results determine the propagation direction of the front for a much broader range of parameters than previous results. This talk is based on joint work with Toshiko Ogiwara (Josai University).

Toshiyuki OGAWA

Toward the global bifurcation structure for traveling wave solutions in three-component competition-diffusion systems

Segregation is one of the important issues in ecology. Here, we will mathematically consider the question of whether an alien species can invade and coexist between two native species. Therefore, we consider the three species competition reaction-diffusion equations of Lotka-Volterra type. Moreover, we consider the situation where an exotic species invades to the buffer zone between the native two strong species. The existence and stability of traveling wave solution connecting two stable constant states in 2-componet strong competition reaction diffusion system is well-known. This traveling wave with no third species is a trivial solution for the 3-component system as well. We are interested in the stability change of this trivial traveling wave solution with respect to the intrinsic growth rate for the third species and study the bifurcation structure around it. Since we have already discussed the local bifurcation in the ReaDiNet workshop before, I am going to briefly review the previous results and discuss global bifurcation structure relating to this problem. This talk is based on the joint works between Chiun-Chuan Chen, Chueh-Hsin Chang, Shin-ichiro Ei, Hideo Ikeda, and Masayasu Mimura.

Sheila Rae PERMANES

Spatio-temporal modeling of host-pathogen interactions using level set method and Fisher-KPP equation

Plant-disease phenotyping is crucial for plant pathologists in the assessment of host-resistance and pathogen aggressiveness. It provides better understanding on pathogen spread within hosts, pathogen evolution and control, and plant breeding for pathogen resistance. Phenotyping is usually performed from visual scoring to image segmentation. In this study, we combine image-based phenotyping with mathematical modeling to improve plant disease quantification by analyzing the spatiotemporal dynamics of plant-pathogen interactions when the pathogen causes growing lesions on plant tissues.

This study utilized a readily available set of images of the spread of Peyronellaea pinodes on the stipules of two pea cultivars that were monitored daily with visible imaging. We implemented the level set method based on the advection equation for the leaf deformation and lesion growth where both follows the same model. We started from defining the computational domain and initializing the level set function which represents the evolving interface, in particular, the boundary of the leaf and the lesion. For the lesion, the boundaries are less sharp and less smooth as appearing in the pixel-segmented images and probability images which makes difficult to detect . Hence, we applied the marching squares to extract the contour of the lesion, and transformed them into binary images in order to implement the level set method. The simulation for both leaf deformation and lesion growth commenced from Day 3 image data and ended at Day 7.

This study provides comparison between the observed image sequences and the solution to the level set method of the two pea cultivars with respect to the leaf deformation and lesion growth through visual representations and metric measures. We show that the mathematical model was able to capture and track the evolution of the leaf deformation and lesion growth with a level of accuracy close to the data images. This comparison can be a critical step in this newly established approach of plant disease phenotyping since it ascertains the method's accuracy and reliability for the mathematical algorithms and techniques used in disease detection and segmentation, supports research validity, and can also contribute to the practical implications for biology, agriculture and similar fields.

Cécile TAING

On the Fisher infinitesimal model without variability

We study the long-time behavior of solutions to a model of sexual populations structured in phenotypes. The model features a nonlinear integral reproduction operator derived from the Fisher infinitesimal operator and a trait-dependent selection term. The reproduction operator describes here the inheritance of the mean parental traits to the offspring without variability. We show that, under assumptions on the growth of the selection rate, Dirac masses are stable around phenotypes for which the difference between the selection rate and its minimum value is less than $1/2$. Moreover, we prove the convergence in some Fourier-based distance of the centered and rescaled solution to a stationary profile under some conditions on the initial moments of the solution. This work has been done in collaboration with Amic Frouvelle

(Univ. Paris Dauphine).

Samuel TRETON

Bridging bulk and surface: an interacting particle system journey towards the field-road diffusion model

This presentation explores the field-road diffusion model developed in 2012 by Berestycki, Roquejoffre, and Rossi. This parabolic system aims to capture the significant dispersal effects induced by lines of fast diffusion, with wide-ranging applications in population dynamics, ecology, and epidemiology.

Initially, we will introduce the model, emphasizing its ability to simulate accelerated spread phenomena. We will then concentrate on the explicit determination of the fundamental solution to the macroscopic system, achieved through the application of a double integral transform, namely Fourier and Laplace. This analytical framework offers clear insights into the model's dynamics and sets the stage for exploring non-linear issues such as "persistence vs. extinction" phenomena in the presence of reaction terms with the so-called Allee effect.

The second part of the talk will be dedicated to provide a stochastic foundation for the deterministic framework by deriving the governing equations of the diffusive field-road model from an interacting particle system. To introduce this approach, we will go back to the origins of the Symmetric Simple Exclusion Process (SSEP) which enables the rigorous derivation of solutions to the Heat equation on the torus. After outlining the principles of this type of particle system, we will see how it can be used to generate certain boundary conditions. This will allow us to introduce a microscopic dynamics for the field-road diffusion model and present our recent result on its hydrodynamic limit.